Reg. No. [| | | | | | | | | | |]

Question Paper Code : 27659

B.E./B.Tech. DEGREE EXAMINATION, DECEMBER 2015/JANUARY 2016

First Semester

Mechanical Engineering

MA6151 : MATHEMATICS – I

(Common to all branches except Marine Engineering)

(Regulations – 2013)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Find the Eigen values of 3A + 2I, where $A = \begin{pmatrix} 5 & 4 \\ 0 & 2 \end{pmatrix}$.
2. What is the nature of the quadratic form $x^2 + y^2 + z^2$ in four variables ?
3. Discuss the convergence of the sequence $\{a_n\}$, where $a_n = \frac{n+1}{n}$.
4. Examine the convergence of the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\ldots$
5. What is the curvature of the circle $(x-1)^2 + (y+2)^2 = 16$ at any point on it ?
6. Define evolutes of the curve.
7. If $x^y + y^x = 1$, then find $\frac{dy}{dx}$
8. If $x = r\cos\theta$, $y = r\sin\theta$, then find $\frac{\partial(x, y)}{\partial(r, \theta)}$.
9. Evaluate $\int_0^{\pi}\int_0^{a} r\,dr\,d\theta$.
10. Sketch the region of integration in $\int_0^1\int_0^x dy\,dx$.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ **(8)**

(ii) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$. Hence find A^{-1}. (8)

OR

(b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by using orthogonal transformation. Hence find its rank and nature. (16)

12. (a) (i) Discuss the convergence and the divergence of the following series :

$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots$ to ∞ (8)

(ii) Find the interval of the convergence $x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \ldots\ldots \infty$ (8)

OR

(b) (i) Examine convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1} - n\right)$. (8)

(ii) Test the convergence of the series $\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + \ldots\ldots$ to ∞ (8)

13. (a) (i) Find the radius of curvature at any point of the catenary $y = c \cosh \frac{x}{c}$. (8)

(ii) Find the equation of the evolutes of the parabola $y^2 = 4ax$. (8)

OR

(b) (i) Find the equation of circle of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$. (12)

(ii) Find the envelope of $y = mx + \sqrt{a^2m^2 + b^2}$, where m is the parameter. (4)

14. (a) (i) Expand $e^x \sin(y)$ in powers of x and y up to the third degree terms. (8)

(ii) If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r \cos\theta$, then find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. (8)

OR

(b) (i) Examine $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values. (8)

(ii) If $w = f(y - z, z - x, x - y)$, then show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$. (8)

15. (a) (i) Evaluate $\iint xy\,dx\,dy$ over the positive quadrant of the circle $x^2 + y^2 = 4$. (8)

(ii) By changing to polar coordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$. (8)

OR

(b) (i) By changing the order of integration evaluation $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$. (8)

(ii) Evaluate $\iiint_V \frac{dz\,dy\,dx}{(x+y+z+1)^3}$ where V is the region bounded by $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$ (8)